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Asymmetry Dynamic Volatility Forecast Evaluations using Interday and Intraday Data

(Penilaian Peramalan Kemeruapan Dinamik Asimetri dengan Data Antara dan Dalaman Harian)

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ABSTRACT

The accuracy of financial time series forecasts often rely on the model precision and the availability of actual observations for forecast evaluations. This study aimed to tackle these issues in order to obtain a suitable asymmetric time-varying volatility model that outperformed in the forecast evaluations based on interday and intraday data. The model precision was examined based on the most appropriate time-varying volatility representation under the autoregressive conditional heteroscedascity framework. For forecast precision, the evaluations were conducted under three loss functions using the volatility proxies and realized volatility. The empirical studies were implemented on two major financial markets and the estimated results are applied in quantifying their market risks. Empirical results indicated that Zakoian model provided the best in-sample forecasts whereas DGE on the other hand indicated better out-of-sample forecasts. For the type of volatility proxy selection, the implementation of intraday data in the latent volatility indicated significant improvement in all the time horizon forecasts.

Keyword: ARCH model; dynamic volatility; market risk; realized volatility

ABSTRAK

Ketepatan ramalan siri masa kewangan sering bergantung kepada ketepatan dan kewujudan cerapan sebenar dalam penilaian ramalan. Kajian ini bertujuan menangani isu-isu tersebut untuk mendapat model kemeruapan berubah masa asimetri yang dapat memberi prestasi yang baik berdasarkan data antara dan dalaman harian. Ketepatan model diperiksa berdasarkan pewakilan kemeruapan berubah masa paling sesuai dengan rangka kerja autoregresi heteroskedastisiti bersyarat. Untuk ketepatan peramalan, penilaian peramalan dijalankan berdasarkan tiga fungsi kerugian dengan proksi kemeruapan dan kemeruapan realisasi. Kajian empirik dilaksanakan pada dua pasaran saham utama dan keputusan penganggaran digunakan dalam mengkuantitikan risiko pasaran masing-masing. Keputusan empirik menunjukkan model asimetri Zakoian memberi keputusan penilaian peramalan dalam sampel yang terbaik manakala model DGE pula menandakan peramalan luar sampel yang paling tepat. Untuk pemilihan proksi kemeruapan, penggunaan data dalaman harian sebagai kemeruapan sebenar menunjukkan pembaikan yang signifikan dalam peramalan semua ufuk masa.

Kata kunci: Kemeruapan dinamik; kemeruapan realisasi; model ARCH; risiko pasaran

INTRODUCTION

In financial time series analysis, volatility forecast is an important topic due to its influential impact in asset pricing modelling, portfolio investment decision as well as risk management development. The introduction of autoregressive conditional heteroscedastic(ARCH) models has successfully captured the clustering volatility especially during the high volatility period in worldwide financial markets. The early volatility representations are dominant by Bollerslev (1986) and Taylor (1986) families of ARCH models. The former family suggested the shocks to variance persist in the form of squared residuals whereas the latter family proposed the shocks in term of absolute residual.

In the further development of ARCH model, Nelson's (1991) later proposed an exponential ARCH to capture the asymmetric news impact (Black 1976) in the return volatility. Sometimes this impact is known as leverage effect where the market volatility tended to rise more in response to bad news as compared to the released of good news. This important stylized fact has been extended in Bollerslev's framework by Glosten et al. (1993) (GJR henceforth) using a dummy variable to capture the impact of bad news. On the other hand, Zakoian (1990) introduced this asymmetric effect using the Taylor's specification. However, both of these specifications have fixed the volatility representation based on the Gaussian assumption where the expected square return is approximated to variance and the expected absolute return is estimated to the standard deviation. Since the worldwide financial markets are deviated from normal distribution with kurtosis exceeded three and non-zero skewness, the aforementioned assumption might not be suitable anymore. Knowing the presence of other potential power form of volatility, Ding, Granger and Engle (DGE henceforth) suggested the asymmetric power ARCH (Ding et al. 1993) with flexible power transformation. This model endogenously estimated the power transformation rather than fixed arbitrarily in GJR and Zakoian models.

Besides the correct model specification, the availability of 'actual' volatility is also an important factor to ensure good forecast performance. In most of the studies, the unobservable actual volatilities often represented by proxies such as square return or absolute return based on the interday data. Consequently, some studies (Jorion 1996; Schwet 1990) reported that the forecast using GARCH model only managed to explain less than 5% to the proxy volatility. The absence of good approximation of actual volatility is the main reason for the poor forecast results. With the fast growing of data base system in financial markets, the high frequency data (intraday data) become available in most of the major financial markets. With the high frequency data (minutely), it is possible to obtain a better approximation of actual volatility. Anderson and Bollerslev (1998) pointed out a better estimation of realized daily volatility can be obtained by summing the 288 5 min squared intraday returns for the 24 h foreign exchange market. This approach is the approximation of a continuous time diffusion process for most of the financial asset prices time series. In addition, it is noted by Ebens (1999) and Anderson et al. (1999) that the squared return is an unbiased estimator but at the same time, a noisy estimator. In the stock market, the intraday returns are obtained by summing the trading hours with the absence of overnight trading. Further information of realized volatility can be obtained from Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002).

The preceding discussions thus provided that the factors underlying the role of superior forecast performance are correct model specification and good estimators of actual volatility. This is an interesting research issue and it is worth exploring on how the DGE, GJR and Zakoian compete in the model specification and forecast performance. In order to do so, this paper attempted to address these issues in two major financial markets, the S&P500 and the FTSE100. The model selections are based on Akiake, Schwert and Hannan-Quinn information criteria. In order to avoid biasness, both the interday and intraday proxies are used in the forecast evaluations.

This paper is organized as follows: the next section describes the data source in term of inter- and intra-day. This is followed by a discussion on the model specifications, estimation, diagnostic and forecast evaluation. This followed by a presentation on the empirical results and application of the estimated results. The last section contains the conclusion of this study.

DATA SOURCE

The empirical data are obtained from two major global stock markets, the S&P500 and FTSE100. The S&P500 is a free-float capitalization-weighted index introduced in year 1957 which traded under the NYSE Euronext and NASDAQ OMX. The S&P Index committees selected the 500 active large-cap common stocks that represented the industries in the United States economy. FTSE100 index was established in year 1984 with 100 most wealthy companies listed on the London Stock Exchange (LSE). These companies contributed approximately 80% of the market capitalization of the LSE. The empirical indices started from January 1998 until December 2008 with a total of 2767 and 2777 observations for S&P500 and FTSE100, respectively. A total of five months daily trading data (102 observations for both markets) are reserved for forecast evaluations. The most common financial data are based on the interday closing prices where daily returns are subsequently calculated. The percentage continuously compounded interday return is defined as

$$R_t = 100(\ln P_t - \ln P_{t-1}). \qquad (1)$$

REALIZED VOLATILITY

The rapid development of recent information and communication technology (ICT) has promoted the use of high frequency data to facilitate a more accurate estimation and forecasting analysis. This referred to intraday data with N observations in one day which are normally recorded in the interval of multiple minute. The percentage of continuously compounded return is defined as:

$$R_{t,a} = 100(\ln P_{t,a-1}), \qquad (2)$$

where $a = 1, \ldots, N$ and $t = 1, \ldots, T$. In other words, each day (t) consists of N recorded trading activities. For this study, the durations for trading hours are from 9:30 to 16:00 (S&P500) and 8:00 to 16:30 (FTSE100) with $N_{\text{S\&P500}}$=390 and N_{FTSE100}=510, respectively. These time series are assumed to have $E[R_{t,a}] = 0$, $\underset{\forall r\neq s, p\neq q}{E}[R_{r,p}\, R_{s,q}] = 0$ and finite $\underset{\forall r,s,p,q}{E}\left[R^2_{r,p}R^2_{s,1}\right]$. For model-free proxy of volatility, the daily squared compounded returns are:

$$\begin{aligned} R_t^2 &= \left[\sum_{a=1}^{N} R_{t,a}\right]^2 \\ &= \sum_{a=1}^{N} R_{t,a}^2 + 2\sum_{a=1}^{N}\sum_{b=1}^{N} R_{t,a}^2 R_{t,b}^2 \left(a \neq b\right). \end{aligned} \qquad (3)$$

The second term indicated the autocovariances which acted as the noise component in the realized volatility. This item vanishes if $\underset{\forall r\neq s, p\neq q}{E}[R_{r,p}\, R_{s,q}] = 0$ and reduced to $E\left[\hat{R}^2_{t,a}\right] = \sum_{a=1}^{N} R^2_{t,a}\sigma^2_{t,actual}$ which is an unbiased estimator

(Anderson et al. 1999) of the daily population variance or latent volatility. The variance of realized volatility can be expressed as:

$$V\left[\hat{R}_t^2\right]=E\left[\sum_{a=1}^{N}\left(R_{t,a}^2-\frac{\sigma_t^2}{N}\right)\right]^2$$
$$=E\left[\sum_{a=1}^{N}\left(R_{t,a}^2-\frac{\sigma_t^2}{N}\right)\right]^2\left(1+2\sum_{a=1}^{N-1}\left(1-\frac{a}{N}\right)\rho_{t,N,a}\right)$$
$$=\left(\sum_{a=1}^{N}E\left[R_{t,a}^4\right]-2\frac{\sigma_t^2}{N}\sum_{a=1}^{N}E\left[R_{t,a}^2\right]+\frac{\sigma_t^4}{N}\right)\left(1+2\sum_{a=1}^{N-1}\left(1-\frac{a}{N}\right)\rho_{t,N,a}\right), \quad (4)$$

under the normality assumption where $N(0,\sigma_t^2/N)$, $V\left[\hat{R}_t^2\right]=2\frac{\sigma_t^2}{N}$ which indicated that the variance of realized volatility reduced at the rate of N.

Methods

Let R_t be a general univariate asset return which is serially uncorrelated but dependent in the ARCH specification. For a given information set I_{t-1} available at time $t-1$, the conditional mean of r_t is defined as:

$$E(r_t|I_{t-1})=E_{t-1}(r_t)=\mu_t, \quad (5)$$

with the innovation process $a_t=r_t-\mu_t$ with the conditional variance $Var(r_t|I_{t-1})=Var_{t-1}(a_t^2)=\sigma_t^2$. In financial time series, the conditional mean often captured by a stationary ARMA(m,n) model under the non-vector form:

$$\mu_t=\phi_0+\sum_{i=1}^{m}\phi_i r_{t-1}+\sum_{i=1}^{n}\theta_i a_{t-1}. \quad (6)$$

An ARCH model is also frequently represented by a regression model in the form of $r_t=x_t'\beta+a_t$ where x_t' is a column vector. The corresponding unconditional variance can be expressed as $Var(a_t(\theta))=E(a_t^2(\theta))=\sigma_t^2(\theta)$ where $E(a_t)=0$ and $E(a_k a_h)=0$ for all $k\neq h$. Further, the conditional variance begun with the relationship $a_t=\sigma_t z_t$ where for standardized process of z_t, $E(z_t|I_{t-1})=0$ and $Var(z_t|I_{t-1})=1$ for all t. Now, consider an asymmetric power DGE GARCH(1,1) model with the following specifications:

$$\sigma_t^{\delta}=\alpha_0+\alpha_1[k_\gamma(a_{t-1})]^{\delta}+\beta_1\sigma_{t-1}^{\delta}, \quad (7)$$

where $k_\gamma(a_{t-1})=|a_{t-1}|-\gamma a_{t-1}$ and δ is the flexible volatility transformation parameter. Specifically, when the conditional volatility representation restricted to $\delta=1$ (conditional standard deviation) and $\delta=2$ (conditional variance), the model changed to Zakovian and GJR models with leverage effect (dummy variable) as follows:

GJR: $$\sigma_t^2=\alpha_0+\alpha_1 a_{t-1}^2+\gamma d_{t-1}a_{t-1}^2\beta_1\sigma_{t-1}^2. \quad (8)$$

Zakoian: $$\sigma_t=\alpha_0+\alpha_1 a_{t-1}+\gamma d_{t-1}a_{t-1}\beta_1\sigma_{t-1}, \quad (9)$$

where $d_{t-1}=\begin{cases}1 & if\ a_{t-1}<0\\ 0 & if\ a_{t-1}>0\end{cases}$. It is worth noting that the GJR asymmetric coefficient initiated with positive sign whereas DGE and Zakoian started with negative sign. This is to make sure that the interpretation of news impact is consistent across the models. For example, $\gamma>0$ indicated the presence of leverage effect with additional impact (γ) as compared to good news. From the economic point of view, the leverage effect can be explained based on the debt-equity ratio. Market equity values often determined by the stock price where a drop in stock price would increased the ratio and consequently increased the risk from the investor perspectives. Thus negative news has a deeper impact to future volatility than positive news.

Maximum Likelihood Estimation

In the maximum likelihood estimation (MLE), z_t normally follows parametric distribution such as normal, student-t and generalized error distribution. Under the assumption of standardized $z_t\sim N(0,1)$, the log-likelihood function with density function is given as:

$$f\left(z_t|\Omega_{t-1}\right)=\left(\frac{1}{2\pi\sigma_t^2}\right)^{\frac{1}{2}}\exp\left(-\frac{1}{2}\frac{a_t^2}{\sigma_t^2}\right) \text{ is}$$

$$L_T(\eta)=\sum_{t=1}^{T}l_t(\eta)=\ln f_a(a_1)$$
$$+\left\{-\frac{T-1}{2}\ln(2\pi)-\frac{1}{2}\sum_{t=2}^{T}\ln\left(\sigma_t^2\right)-\frac{1}{2}\sum_{t=2}^{T}\frac{a_t^2}{\sigma_t^2}\right\}, \quad (10)$$

where $\eta=(\alpha_0, \alpha_1, \beta_1, \gamma, \delta)$ represents the vector of unknown parameter for conditional dispersion equation all set at time t. For large sample size, the unknown marginal density log $f_a(a_1)$ can be ignored under the following derivation:

$$L_T(\eta)=\sum_{t=1}^{T}\ln f(a_T,\ldots,a_2|a_1)=\sum_{t=2}^{T}\ln f(a_t|\Omega_{t-1}). \quad (11)$$

Apart from the constants, $l_t(\eta)=-\frac{1}{2}\ln\left(\sigma_t^2\right)-\frac{1}{2}\frac{a_t^2}{\sigma_t^2}$. Differentiating with respect to the vector parameter yields:

$$\frac{\partial l_t}{\partial\eta}=-\frac{a_t}{\sigma_t^2}\frac{\partial a_t}{\partial\eta}-\frac{1}{2}\left(\frac{1}{\sigma_t^2}-\frac{a_t^2}{\sigma_t^4}\right)\frac{\partial\sigma_t^2}{\partial\eta}. \quad (12)$$

However, the DGE model is computed under the representation of σ_t^{δ}, therefore the additional separated analytical derivatives for conditional dispersion are

$$\frac{\partial\sigma_t^2}{\partial\theta}=-\frac{2\sigma_t^2}{\delta\sigma_t^{\delta}}\frac{\partial\sigma_t^{\delta}}{\partial\theta} \text{ and}$$

$$\frac{\partial\sigma_t^2}{\partial\delta}=-\frac{2\sigma_t^2}{\delta\sigma_t^{\delta}}\left[\frac{1}{\delta}\frac{\partial\sigma_t^{\delta}}{\partial\delta}-\frac{\sigma_t^{\delta}}{\delta^2}\ln\sigma_t^{\delta}\right], \quad (13)$$

where $\theta = (\alpha_0, \alpha_1, \beta_1, \gamma)$. The vector gradients with respect to the conditional dispersion parameter can be obtained in the following equations:

$$\frac{\partial\sigma_t^\delta}{\partial\alpha_t} = k_\gamma\left(a_{t-1}\right)^\delta + \beta_1\frac{\partial\sigma_{t-1}^\delta}{\partial\alpha_1};$$

$$\frac{\partial\sigma_t^\delta}{\partial\gamma} = \alpha_1\delta k_\gamma\left(a_{t-1}\right)^{\delta-1}a_{t-1} + \beta_1\frac{\partial\sigma_{t-1}^\delta}{\partial\gamma};$$

$$\frac{\partial\sigma_t^\delta}{\partial\delta} = \alpha_1\left[k_\gamma\left(a_{t-1}\right)^\delta \ln k_\gamma\left(a_{t-1}\right)\right] + \beta_1\frac{\partial\sigma_{t-1}^\delta}{\partial\delta}. \tag{14}$$

A more comprehensive analytic derivatives of DGE APARCH(p,q) can be found in Laurent (2004) and He and Terasvirta (1997). In this study, the Gaussian Quasi Maximum Likelihood (Bollerslev & Wooldridge 1992) method is used to provide consistent (at least) estimation under the correct specification as stated earlier even under the non-normal condition of z_t. For faster and easier computation, we have selected the Marquardt (1963) method where only the outer products of the gradient vectors are computed in the iterative estimations:

$$\eta^{(k+1)} = \eta^{(k)} + \left(\sum_{t=1}^{T}\frac{\partial l_t^{(k)}}{\partial\eta}\frac{\partial l_t^{(k)}}{\partial\eta'} - cl\right)^{-1}\frac{\partial L_T^{(k)}}{\partial\eta}, \tag{15}$$

where cl is a constant diagonal matrix. This correction matrix provided better maximum location identification by following the direction of the gradient vector. Under the regularity conditions of quasi maximum likelihood estimation (QMLE), the large number samples asymptotically normally distributed $\hat{\eta}$ following the property:

$$\sqrt{T}\left(\hat{\eta} - \eta_{true}\right) \xrightarrow{D} N\left(0, \hat{J}^{-1}\hat{I}\hat{J}^{-1}\right), \tag{16}$$

where $\hat{J} \equiv \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\left(E\left[\frac{\partial^2 l_t(\eta)}{\partial\eta\partial\eta'}\middle|\Omega_{t-1}\right]\right)$

and $\hat{I} \equiv \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}E\left[\left(\frac{\partial^2 l_t(\eta)}{\partial\eta}\right)\left(\frac{\partial l_t(\eta)}{\partial\eta}\right)'\middle|I_{t-1}\right].$

The QMLE become ordinary MLE if the z_t is truly normally distributed where the $J^{-1}IJ^{-1}$ reduced to J^{-1} which is $\sqrt{T}\left(\hat{\eta} - \eta_{true}\right) \xrightarrow{D} N\left(0, \hat{J}^{-1}\right)$ under the asymptotic property of MLE.

However, the non-normality (fat-tail property) of financial time series is often observed in the worldwide financial markets. Although normality assumption ML estimator may fulfil the consistency condition, the departure from normality on the other hand can cause inefficient issue in the estimations. Thus, to circumvent the leptokurtosis ARCH issue, Bollerslev (1987) introduced the heavy tail standardized student-t with degree of freedom exceeded 2 in the univariate time series. The student-t distribution (υ) can be written as

$$f(z_t;\upsilon|\Omega_{t-1}) = \frac{\Gamma\left[\frac{(\upsilon+k)}{2}\right]}{\left[\pi(\upsilon-2)\right]^{k/2}\Gamma\left[\frac{(\upsilon)}{2}\right]}\left[1+\frac{z_t^2}{(\upsilon-2)}\right]^{-(\upsilon+k)/2}. \tag{17}$$

The associated log-likelihood function can be expressed as:

$$l_t(\Theta) = \ln\left\{\frac{\Gamma\left[\frac{(\upsilon+k)}{2}\right]}{\left[\pi(\upsilon-2)\right]^{k/2}\Gamma\left[\frac{\upsilon}{2}\right]}\right\} - \frac{1}{2}\ln\sigma_t^2 - \frac{1}{2}(\upsilon+k)\ln\left[1+\frac{a_t^2\sigma_t^2}{(\upsilon-2)}\right]. \tag{18}$$

DIAGNOSTIC AND MODEL SELECTION

For model diagnostic, the Ljung-Box serial correlation and Engle ARCH tests are used to examine the standardized and squared standardized residuals under the null hypothesis that the noise terms are serially uncorrelated or random. Model selections are based on the Akaike information criterion (AIC), Schwert information criterion (SIC) and Hannan-Quinn information criterion (HIC) which evaluated from the adjusted (penalty function due to additional number estimated parameters) average log likelihood function (LT) are selected for the estimation evaluation. The information criteria can be expressed as:

$$AIC = -2\,{}^{L_T}\!/_{T} + 2\,{}^{k}\!/_{T}.$$

$$SIC = -2\,{}^{L_T}\!/_{T} + 2\,{}^{\ln(k)}\!/_{T}.$$

$$HIC = -2\,{}^{L_T}\!/_{T} + 2\,{}^{k\ln[\ln(T)]}\!/_{T}. \tag{19}$$

where k is the number of estimated parameters.

FORECAST EVALUATION

For out-of-sample one-day-ahead forecasts, each volatility model is estimated H times based on fix period of T observations. In forecast evaluations, the mean square error (MSE), the root mean square error (RMSE) and the mean absolute (MAE) are calculated as follows:

$$\text{MSE} = \frac{1}{H}\sum_{t=T+1}^{T+H}\left(actual_t - forecast_t\right)^2.$$

$$\text{RMSE} = \frac{1}{H}\sqrt{\sum_{t=T+1}^{T+H}\left(actual_t - forecast_t\right)^2}.$$

$$\text{MAE} = \frac{1}{H}\sum_{t=T+1}^{T+H}\left|actual_t - forecast_t\right|; \tag{20}$$

where the *actual* and *forecast* represented three forms of volatility proxies with $|r_t|$, r_t^2 and minutely realized volatility to avoid the possible biasness in the forecast evaluations. Finally, the Mincer-Zarnowitz(1969) regression is used to further evaluate the relationship between the forecast and actual (proxy) based on the coefficient of determination, R^2 as follow:

$$\sigma^2_{t,actual} = \lambda_0 + \lambda_1 \sigma^2_{t,forecast} + u_t . \quad (21)$$

Conditioning upon the forecast, the forecast is unbiased and optimal only if $\lambda_0 = 0$ and $\lambda_1 = 1$ knowing that the conditional mean is zero. The determinant coefficient, R^2 indicated the power of predictability of the selected models with R^2_{interday} and R^2_{intraday}. More specifically, the R^2 expressed the proportion (percentage) of the total variation in the actual values that can be accounted for a linear relationship with the forecast value.

EMPIRICAL RESULTS

In order to examine the presence of fat-tailed property, quantile-quantile plots are conducted for S&P500 and FTSE100. Figures 1 and 2 indicate both the indices deviated from a normal distribution (heavier at both tails), however, fitted better after replacing by a student-t distribution. In other words, a heavy-tail distribution should be considered in the model specification. For conditional mean equation analysis, a moving average MA(1) model is capable of adjusting the serial correlation in the S&P500 while an ARMA(1,1) is needed for FTSE100. According to Miller et al. (1994), similar correction can be done using autoregressive model.

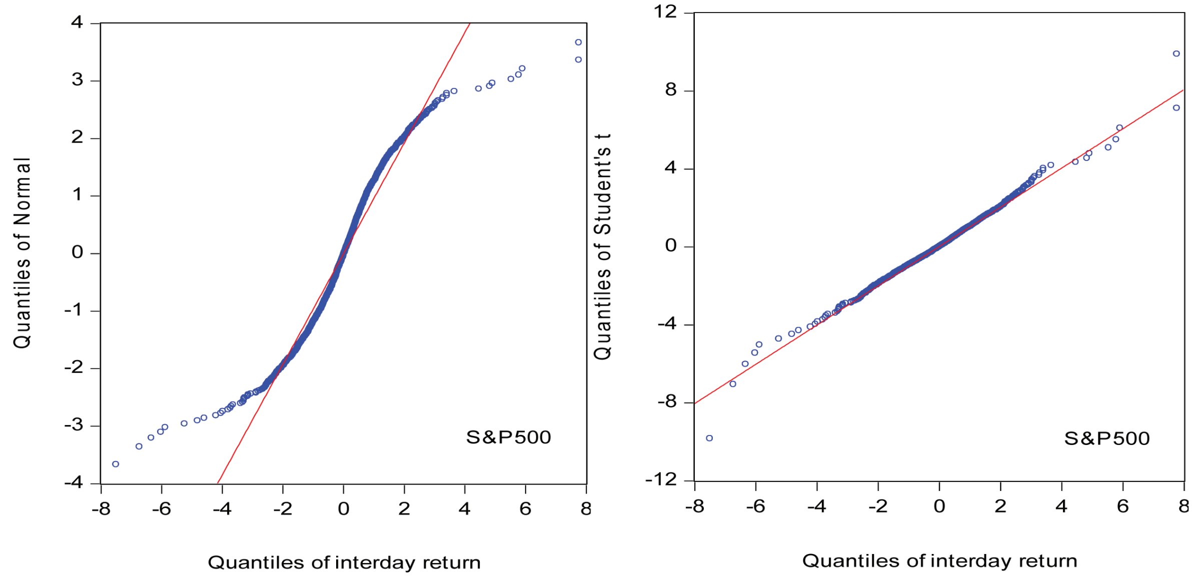

FIGURE 1.Quantile-quantile plots for S&P500

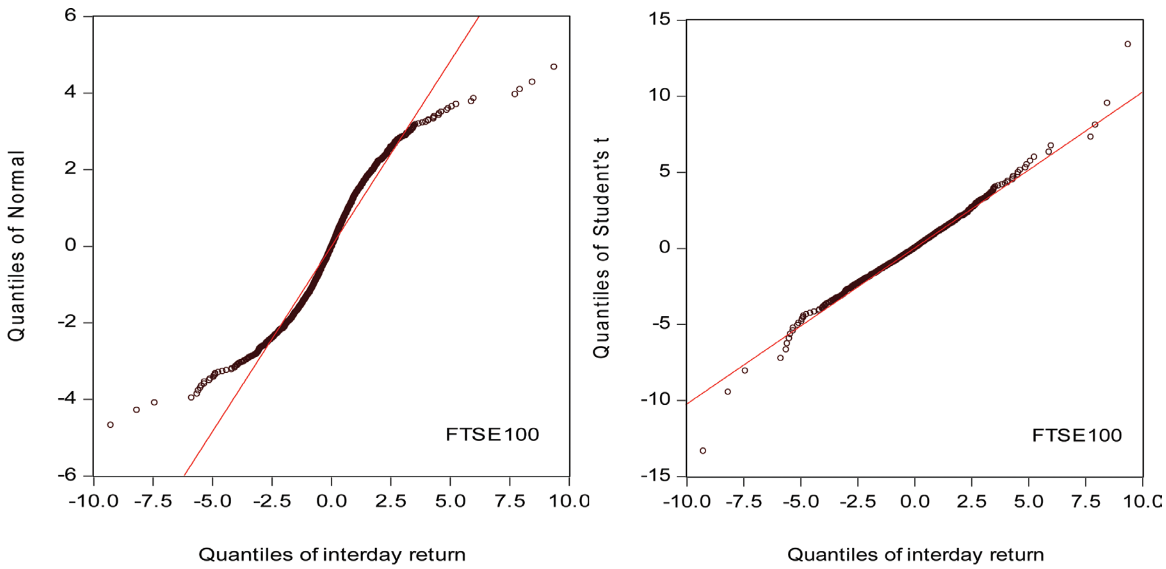

FIGURE 2.Quantile-quantile plots for FTSE100

ESTIMATION RESULT

For conditional volatility, the coefficients that directly governed the behaviour of the dynamic volatility modelling can be elaborated as follows:

S&P500 Index. The degree of freedom vs for student-t distributions are all statistically significant in all the models with the values approximately to 20. These results are far from the expectation for v which is range from three to six (Bollerslev 1987). With the high value of v, one expects that the student-t distribution approximately follows a normal distribution. These are evidenced from the results in Table 1 where both different distribution assumptions indicated similar estimation outcomes. Next, the power transformation coefficient δs are close to unity with values 1.040740 and 1.037535 in the DEG-normal and DEG-t. Using t-test, the δs are both statistically different from two but not from unity. These findings implied that the representation of conditional standard deviation is more suitable than conditional variance. In short, the volatility representation is in favour of Zakoian specification in this specific case. On the other hand, the δs are fixed as one and two for Zakoian and GJR model. The volatility persistent can be observed from the coefficient β_1. It is found that the β_1s are less persistence in GJR than DGE and Zakoian models in both the normal and student-t assumptions. The summation of α and β for the Bollerslev GARCH model normally indicates the volatility persistence. However, this is not exactly the same for the APARCH specification under the additional asymmetric effect and power transformation in the conditional volatility.The volatility persistence is less intense when the power transformation increased from 1 to 2. These findings are similar to Ding et al. (1993) where the absolute return exhibited longer memory than the squared returns. In short, higher persistence implied higher correlation between the current and historical volatility. Some studies (Cheong et al. 2007; McMillan & Thupayagale 2008) even included this measurement as the predictability component which provided further implication against the efficient market hypothesis (Fama 1998). For leverage effect, the γs are positive and statistically different from zero at 5% significant level in all the models. This implied that downward movements (shock) in the S&P500 market are followed by greater volatilities than upward movements of the same magnitude. Under the ordinary market condition, this can be easily explained by using the leverage ratio (similar to debt-equity ratio) (Black 1976) of an industry where a crash in stock price can lead to an increase in equity risk and thus triggered a more intense volatility. An interesting finding is also noted from the S&P500 where there is a drastic reduction impact of leverage effect when the power coefficient switched from unity to two where DGE and Zakoian indicated γ closed to one whereas GJR only indicated value close to 0.2000. In other words, the news impact is less sensitive to the square of shock (a_t^2) than (a_t).

FTSE100. Similar analysis has been conducted on the FTSE100. The vs (degree of freedom) for student-t distributions are approximately equal to 16 which are slightly heavier than S&P500. However, these results are still far larger than the range three to six. Thus, Table 2 indicated similar results as S&P500 where the models with both normal and student-t assumptions indicated similar estimation outcomes. The power transformations based on δs are less than unity with values 0.798110 and 0.894708 in the DEG-normal and DEG-t. Again the DEG volatility representation is in favour of standard deviation. These values are slightly lower than the S&P500 index. For volatility persistence, β_1s are slightly less persistence in GJR than DGE and Zakoian models in both the normal and student-t assumptions. These results once again suggested that the absolute conditional standard deviation is more persistence as compared to conditional variance. The news impact coefficient, γs are positive and implied the presence of leverage effect. Similar results are observed where the impact of leverage effect reduced when the power coefficient switched from unity to two.

Overall, both the markets suited better in the Zakoian specification. However, the models based on student-t and normal assumptions are almost identical based on the information criteria due to the large value of degree of freedom.

DIAGNOSTIC AND MODEL SELECTION

For S&P500, only the DGE and Zakoian models failed to reject the null hypothesis of randomness under the Ljung-Box serial correlation for standardized and squared standardized residuals. However, the GJR model indicated the presence of serial correlation in the squared standardized residuals at 10% level of significance in both normal and student-t assumptions. In other words, the volatility representation in term of conditional variance is statistically less suitable in the model specification. In FTSE100, all the models successfully passed the diagnostic tests. Tables 1 and 2 illustrate the results for both markets.

Next, the model selection can be firstly seen from their log likelihood functions (L_T) in both the markets. Overall, L_T (in term of magnitude) decreased from DGE, Zakoian and finally GJR models. Based on the L_T, the DGE models are expected to outperform than other models. However, DGE model has disadvantage over the information criteria evaluation due to one additional estimated parameter as compared to other two models. As a result, the Zakoian model indicated the smallest information criteria for AIC, SIC and HIC. This followed by DGE and finally GJR models. On the other hand, the models based on student-t assumption indicated slightly better information criteria evaluations than normally distributed residual. In short, the Zakoian student-t models are selected as the appropriate models in both the markets for in-sample estimation. This is followed by DGE student-t and finally the GJR student-t models. However, the information criterion only indicated

TABLE 1. Maximum likelihood estimation of S&P500

Estimation	APARCH-normal		APARCH-t		Zakoian-N		Zakoian-t		GJR-N		GJR-T	
	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat
θ_0	-0.006436	-0.390264	0.002625	0.165444	-0.007155	-0.432069	0.001780	0.112571	0.003600	0.223780	0.012107	0.761349
θ_1	0.331351*	17.90905	0.338074*	18.36580	0.331663*	17.95194	0.338263*	18.42660	0.322425*	17.22919	0.332607*	17.97529
α_0	0.013695*	4.112626	0.011684*	5.020679	0.013931*	4.248052	0.011932*	5.754518	0.008859*	3.358330	0.007024*	4.672751
α_1	0.073579*	3.682789	0.073322*	5.435643	0.074543*	5.115195	0.073708*	8.439385	-0.001246	-0.072190	-0.005063	-0.575494
γ	0.994025*	2.822772	0.999999*	4.378756	0.983426*	3.912632	0.999999*	8.706790				
γ^*	0.146734		0.146643		0.147850		0.147416		0.183796*	8.361579	0.183596*	9.916946
β_1	0.923861*	74.90900	0.926333*	113.5827	0.924707*	74.30300	0.927495*	120.0658	0.901041*	64.88132	0.907474*	88.17614
δ	1.040740*	6.312155	1.037535*	6.446784								
ν			20.93018*	4.440399			20.92596*	4.444427			19.00697*	4.366151
Selection												
L	-3121.997		-3113.432		-3122.041		-3113.473		-3134.673		-3125.415	
AIC	2.261653		2.256185		2.260962		2.255492		2.270093		2.264124	
SIC	2.276644		2.273317		2.273811		2.270483		2.282942		2.279114	
HQC	2.267067		2.262373		2.265603		2.260906		2.274733		2.269538	
Diagnostic												
$\tilde{a}_t$ Q(6)	8.3251	0.139	8.5951	0.126	8.3705	0.137	8.6581	0.124	7.9647	0.158	7.6006	0.180
$\tilde{a}_t^2$,Q(6)	9.1769	0.102	8.5273	0.129	9.1040	0.105	8.5038	0.131	10.828	0.055	9.7263	0.083
ARCH(6)	1.521103	0.1670	1.415468	0.2046	1.510492	0.1705	1.414953	0.2048	1.816526	0.0920	1.631498	0.1343

Notes:

1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test (Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis – No serial correlation;LM ARCH test: Null hypothesis - No ARCH effect;
2. The values in the parentheses represents the robust standard error and p-value for estimation and diagnostic respectively.
3. * denotes significant at 5% level.

TABLE 2. Maximum likelihood estimation of FTSE100

Estimation FIGARCH	APARCH-normal		APARCH-t		Zakoian-N		Zakoian-t		GJR-N		GJR-T	
	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat	coefficient	z-stat
θ_0	-0.002376	-0.155013	0.008710	0.575185	-0.000298	-0.019501	0.009875	0.653427	0.007592	0.520839	0.018887	1.305548
θ_1	0.661351*	3.652355	0.663925*	3.621964	0.652163*	3.585520	0.661172*	3.632209	0.649809*	4.434064	0.674138*	4.759545
θ_2	-0.699772*	-4.057454	-0.701442*	-4.009860	-0.692195*	-4.006846	-0.699624*	-4.032060	-0.701326*	-5.129179	-0.723006*	-5.469364
α_0	0.015707*	5.615806	0.015495*	5.643371	0.015515*	5.567858	0.015280*	5.585164	0.013848*	4.727636	0.012918*	4.692285
α_1	0.065441*	7.031403	0.064418*	7.637361	0.065029*	6.152406	0.062694*	6.845686	0.010539	0.635998	0.000947	0.081894
γ	0.861357*	4.973619	0.953704*	7.047430	0.832021*	3.665105	0.964962*	5.556657	0.117916*	4.834397	0.131879*	8.334107
β_1	0.938109*	111.2668	0.937759*	116.9754	0.934574*	103.6161	0.936146*	113.7417	0.918402*	90.36389	0.920864*	89.20045
δ	0.798110*	4.692482	0.894708*	5.459174								
v			16.98173*	4.369429			16.77385*	4.449978			15.46000	4.502544
Selection												
L	-4022.203		-4011.723		-4022.750		-4011.872		-4032.456		-4020.266	
AIC	2.903605		2.896775		2.903278		2.896162		2.910271		2.902209	
SIC	2.920691		2.915996		2.918228		2.913247		2.925221		2.919295	
HQC	2.909775		2.903716		2.908677		2.902332		2.915670		2.908379	
Diagnostic												
$\tilde{a}_t$ Q(6)	5.7150	0.221	5.9942	0.200	5.7266	0.221	5.9550	0.203	5.7126	0.222	5.3617	0.252
$\tilde{a}_t^2$,Q(6)	2.3727	0.668	2.6975	0.610	1.8285	0.767	2.3964	0.663	1.4534	0.835	1.0414	0.791
ARCH(6)	0.392854	0.8840	0.448080	0.8467	0.304995	0.9346	0.398354	0.8805	0.245097	0.9614	0.259965	0.9554

Notes:

1. $\tilde{a}_t$ represents the standardized residual. Ljung Box Serial Correlation Test (Q-statistics) on $\tilde{a}_t$ and $\tilde{a}_t^2$: Null hypothesis – No serial correlation;LM ARCH test: Null hypothesis - No ARCH effect;
2. The values in the parentheses represented the robust standard error and p-value for estimation and diagnostic respectively.
3. * denoted significance at 5% level.

marginal improvement for DGE over Zakoian models. As a summary, the conditional volatility representation and also the distribution assumption played important roles in determining the estimation performance under the information criteria.

OUT-OF-SAMPLE FORECAST EVALUATIONS

It is important to note that superiority in-sample estimation does not guarantee similar out-of-sample forecasts. Due to this, $|r_t|$, r_t^2 and RV have been selected as the volatility proxies to examine the one-step-ahead out-of-sample forecasts for duration from Jan 2009 to May 2009 with a total of 102 trading days. The out-of-sample forecast evaluations have been divided into monthly, three months and five months time horizon using MSE, RMSE and MAE. Tables 3 and 4 present all the forecast evaluations where the loss function for MSE indicated the largest values, followed by RMSE and lastly the MAE in all the time horizons. These findings evidenced similar argument by Andersen (1999) where squared return (volatility proxy) is an unbiased but less efficient estimator for the latent volatility. When the square-root applied to MSE, the RMSE indicated the reduction by the power of half. Lastly, the MAE indicated smallest magnitude since the MSE is relatively more sensitive to extreme value.

From Tables 3 and 4, it is quite obvious that the selection of volatility proxy is an important step to determine the forecast evaluation results. In general, the DGE scored the lowest loss function results, followed by the Zakoian model and finally the GJR models in time-horizon of all the forecast evaluation. Again, superiority in-sample estimation does not always guarantee similar out-of-sample forecasts. However, the scores for DEG is slightly better than GJR models. From MSE, RMSE and MAE, the realized volatility indicated smallest values among others in all the time-horizons. Among the models based on normal and student-t assumption, the forecast performances are not consistent across the forecast time-horizons and proxy selections. For example in both S&P500 and FTSE100, the DGE-t of 20 one-step-ahead forecasts using realized volatility is better than DGE-n, whereas the opposite results are observed when changed to other volatility proxies (absolute and square return). For further evaluation, the Mincer-Zarnowitz regression is applied on the forecasts and proxy volatility. As expected, Tables 5 and 6 indicate that the R^2 for S&P500 and FTSE are approximately 0.40 and 0.20 among the realized volatility and empirical forecasts. In other words, approximately 40% and 20% of the forecasts in S&P500 and FTSE are able to explain the total variation in the realized volatility. For proxies using absolute and square returns, the same forecasts are managed to describe 1.5% to 3.0% for both the S&P500 and FTSE100, respectively. These findings are in concordance with Jorion (1996) where the R^2 only managed to account for less than 5% in most of the GARCH models. However, the poor results are actually due to the absence of 'actual' volatility (Anderson & Bollerslev 1998). When the realized volatility is used, a drastic improvement is observed in the forecast performance. As a result, the correct choice of volatility proxy can strongly influence the forecast performance, for this specific case the realized volatility with minutely data for S&P500 and FTSE100. Figures 3 and 4 illustrate

TABLE 3. The S&P500 Out-of-sample forecasts

Volatility proxies (1 month)	MSE			*RMSE*			MAE		
	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t
APGARCH-n	3.415782	5.965312	1.323124	1.848183	2.442399	1.150271	1.538009	2.205951	0.843671
APARCH-t	3.847131	6.098737	1.272561	1.961411	2.469562	1.128079	1.649954	2.221888	0.824917
Zakoian GARCH-n	3.363442	5.944695	1.335174	1.833969	2.438174	1.155497	1.524788	2.202501	0.845314
Zakoian GARCH-t	3.807993	6.080742	1.274935	1.951408	2.465916	1.12913	1.640926	2.218638	0.824737
GJR GARCH-n	3.825068	6.011325	1.397762	1.955778	2.4518	1.18227	1.640569	2.230233	0.870997
GJR GARCH-t	4.448474	6.213525	1.428514	2.109141	2.492694	1.195205	1.819188	2.247107	0.863794
(3 month)	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t
APGARCH-n	4.398195	12.79528	1.845387	2.097187	3.577049	1.35845	1.688539	2.833685	1.097882
APARCH-t	4.804143	12.79771	1.831918	2.191835	3.577388	1.353484	1.771467	2.847205	1.105915
Zakoian GARCH-n	4.323784	12.7943	1.851526	2.079371	3.576912	1.360708	1.674005	2.831524	1.097182
Zakoian GARCH-t	4.750514	12.79743	1.834903	2.179567	3.57735	1.354586	1.761276	2.845914	1.106221
GJR GARCH-n	5.370283	12.74821	1.910784	2.317387	3.570463	1.382311	1.897214	2.854729	1.119208
GJR GARCH-t	5.849683	12.76557	1.952537	2.418612	3.572894	1.397332	2.015819	2.870256	1.129545
(5 month)	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t	$\|r_t\|$	r_t^2	RV_t
APGARCH-n	3.016414	10.56699	1.3026	1.736783	3.250691	1.141315	1.323244	2.351183	0.855448
APARCH-t	3.282645	10.55269	1.289166	1.811807	3.248491	1.135415	1.38245	2.363584	0.86002
Zakoian GARCH-n	2.971271	10.56572	1.306497	1.723738	3.250496	1.143021	1.314665	2.349511	0.855128
Zakoian GARCH-t	3.246844	10.55206	1.291605	1.8019	3.248394	1.136488	1.375023	2.361691	0.860288
GJR GARCH-n	3.675747	10.56141	1.341417	1.917224	3.249832	1.158196	1.476027	2.387118	0.875671
GJR GARCH-t	3.975100	10.5531	1.361654	1.993765	3.248554	1.166899	1.551772	2.397888	0.879962

TABLE 4. The FTSE Out-of-sample forecasts

Volatility proxies (1 month)	MSE			*RMSE*			MAE		
	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t
APGARCH-n	7.02228	42.58578	0.845977	2.649959	6.525778	0.91977	2.268005	4.543754	0.698862
APARCH-t	6.475978	42.21017	1.029500	2.544794	6.496935	1.014643	2.198906	4.481609	0.772714
Zakovian GARCH-n	6.969595	42.49303	0.967884	2.639999	6.518668	0.983811	2.274463	4.556744	0.756864
Zakovian GARCH-t	7.324815	42.74687	0.814513	2.706439	6.538109	0.902504	2.312542	4.587298	0.685012
GJR GARCH-n	10.07743	43.91994	1.053146	3.174496	6.627212	1.026229	2.751085	4.871820	0.903082
GJR GARCH-t	10.58161	44.26231	0.992623	3.252938	6.652993	0.996304	2.814154	4.915480	0.859020
(3 month)	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t
APGARCH-n	9.342475	42.34543	2.702832	3.056546	6.507337	1.644029	2.541236	4.523926	1.207859
APARCH-t	8.452437	42.41869	2.375767	2.907307	6.512963	1.541352	2.436632	4.496015	1.157084
Zakovian GARCH-n	8.835512	42.5128	2.439173	2.972459	6.520184	1.561785	2.48995	4.525207	1.169497
Zakovian GARCH-t	9.563777	42.39708	2.740579	3.092536	6.511304	1.65547	2.571458	4.540385	1.211343
GJR GARCH-n	11.11762	43.23149	2.958147	3.33431	6.575065	1.719926	2.815727	4.677997	1.339611
GJR GARCH-t	11.80213	43.18115	3.232554	3.435423	6.571236	1.79793	2.894782	4.696496	1.369139
(3 month)	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t	$\lvert r_t \rvert$	r_t^2	RV_t
APGARCH-n	6.616367	31.39332	2.691125	2.57223	5.602974	1.640465	2.032906	3.747266	1.159328
APARCH-t	6.093641	31.42818	2.450103	2.46853	5.606085	1.56528	1.980517	3.736334	1.113556
Zakovian GARCH-n	6.355489	31.4571	2.455324	2.521009	5.608663	1.566947	2.022223	3.759206	1.112608
Zakovian GARCH-t	6.76209	31.40833	2.697493	2.600402	5.604314	1.642405	2.055264	3.759524	1.157399
GJR GARCH-n	8.000593	31.75564	2.616511	2.828532	5.635214	1.617563	2.288569	3.883655	1.191599
GJR GARCH-t	8.346063	31.72286	2.822943	2.888955	5.632305	1.680162	2.314249	3.885762	1.217204

the conditional return and volatility forecasts for both the S&P500 and FTSE100.

APPLICATION IN VALUE-AT-RISK

One of the immediate applications of estimated conditional volatility is quantifying the market risk of a particular financial market. For example, an investor is holding a long trading position $1 million of a stock. His worst loss in next day under normal market condition can be determined using value-at-risk (Jorion 2001). The VaR is defined as the worst loss for a given confidence level (for instance 95%) means one is 95% certain that at the end of a chosen risk horizon (one day ahead for this specific study), there will be no greater loss than VaR under normal market conditions. In portfolio analysis, the VaR often acted as a tool to alert investors for their possible expose risks under a particular portfolio.

Consider the estimated values for S&P500 market using DGE-normal at $t = 2767$ (31 Dec 08) are $r_{2767} = 2.150185738$ and $\sigma^2_{2767} = 3.103242565$, respectively. Thus, the one-day-ahead forecasts are $\hat{r}_{2767}(1) = 0.48396979$ and $\hat{\sigma}^2_{22767}(1) = 2.7090862$. For lower tail 5% quantile, the value is $\hat{r}_{2767}(1) + z_{\alpha=0.05} \times \hat{\sigma}^2_{22767}(1) = 0.4839697 \,||\, 1.6448536 \times 2.7090862 =$ -3.9722% (negative sign indicated the loss). The 95% VaR for a position long of $1 million is $1million × 3.9722% = $39722 with the condition the parameters in the model still holds. In other words, with 95% confidence the potential loss of holding this position in next day is $39722 or less.

Similarly, the lower 5% quantile using DGE-t can be determined as $\hat{r}_{2767}(1) + t_{\alpha=0.05,(v=20.93018)} \times \hat{\sigma}^2_{22767}(1) =$ $0.4951189 \,||\, 2.8856439 \times 2.0859634 = -5.5242\%$. The heavy-tail assumption increased the loss to $55242 as compared to $39722 in the normal distribution model. It is worth to note that the VaR is directly influenced by the parametric distribution assumption. For this specific study we use normal and student-t distribution. In other words, the value of VaR is varied based on the behaviour of the tail distribution time series.

CONCLUSION

This study investigated the importance of volatility representation and choices of volatility proxy in forecast evaluations. First, three volatility representations namely the DGE, Zakoian and GJR specifications have been used in order to obtain the most appropriate in-sample forecast for the S&P500 and FTSE100 markets. Although the in-sample forecasts (estimation) are in favour of Zakoian models using three loss functions, the out-of-sample forecast on the other hand indicated DGE models provided the lowest forecast evaluation results. However, the improvement of forecasts is in marginal form which implied that superiority in-sample forecast does not always guarantee out-of-sample forecasts.The second important issue in forecast is the availability of actual observations. In this specific study, the volatility proxies are absolute return, squared return and high frequency realized volatility. Overall, when the realized volatility acted as the unobserved latent volatility, the out-of-sample forecasts indicated drastic improvement in all the time horizons.

As a conclusion, an accurate model specification must complement by the availability of 'actual' value in forecast evaluations. For this specific study, a dynamic conditional variance model evaluated using minutely realized volatility.

TABLE 5. Mincer-Zarnowitz regression test for S&P500

Volatility proxies (5 months)	$\|r_t\|$			r_t^2			RV_t		
	a	*b*	R^2	a	*b*	R^2	a	*b*	R^2
APGARCH-n	1.042353**	0.130715	0.026403	1.556225**	0.478799*	0.029578	0.733799**	0.788745**	0.412032
APARCH-t	1.034373**	0.129216*	0.027376	1.536032**	0.469727*	0.030205	0.720646**	0.765831**	0.412148
Zakovian GARCH-n	1.041183**	0.131743	0.026257	1.555555**	0.481070*	0.029232	0.726162**	0.795181**	0.409990
Zakovian GARCH-t	1.033650**	0.130009*	0.027430	1.537329**	0.471046*	0.030065	0.721093**	0.768645**	0.410944
GJR GARCH-n	1.050609**	0.118706	0.026370	1.513946**	0.462590*	0.033436	0.794068**	0.712278**	0.406925
GJR GARCH-t	1.033497**	0.120958*	0.028841	1.468766**	0.463411**	0.035344	0.772414**	0.695820**	0.409054

TABLE 6. Mincer-Zarnowitz regression test for FTSE100

Volatility proxies (5 months)	$\|r_t\|$			r_t^2			RV_t		
	a	*b*	R^2	a	*b*	R^2	a	*b*	R^2
APGARCH-n	1.065877**	0.108427	0.019422	1.458590	0.667081*	0.036082	1.959180**	0.379364**	0.181513
APARCH-t	1.059720**	0.112193	0.017387	1.384448	0.701989*	0.033411	1.877685**	0.411950**	0.178967
Zakovian GARCH-n	1.066408**	0.108157	0.016743	1.421807	0.678165*	0.032309	1.852237**	0.413045**	0.186426
Zakovian GARCH-t	1.068189**	0.106710	0.019142	1.471243	0.657013*	0.035617	1.945322**	0.380284**	0.185601
GJR GARCH-n	1.080881**	0.093425	0.014402	1.582779	0.565630	0.025912	1.705296**	0.414845**	0.216806
GJR GARCH-t	1.077899**	0.093777	0.016360	1.579915	0.563417*	0.028986	1.779488**	0.391435	0.217620

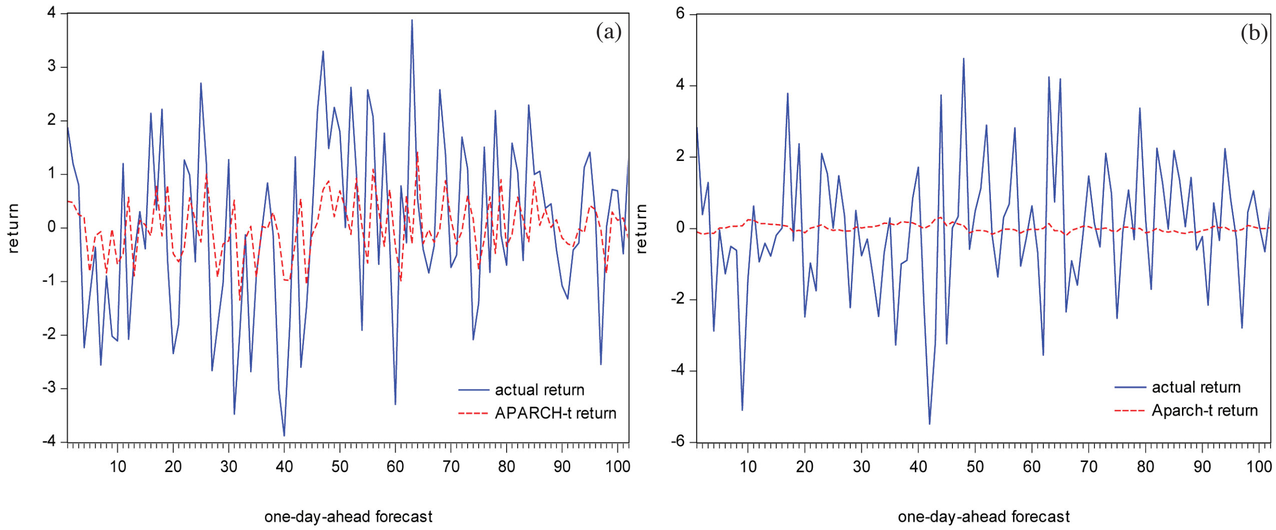

FIGURE 3. 5 months return forecast results for (a) S&P500 and (b) FTSE100

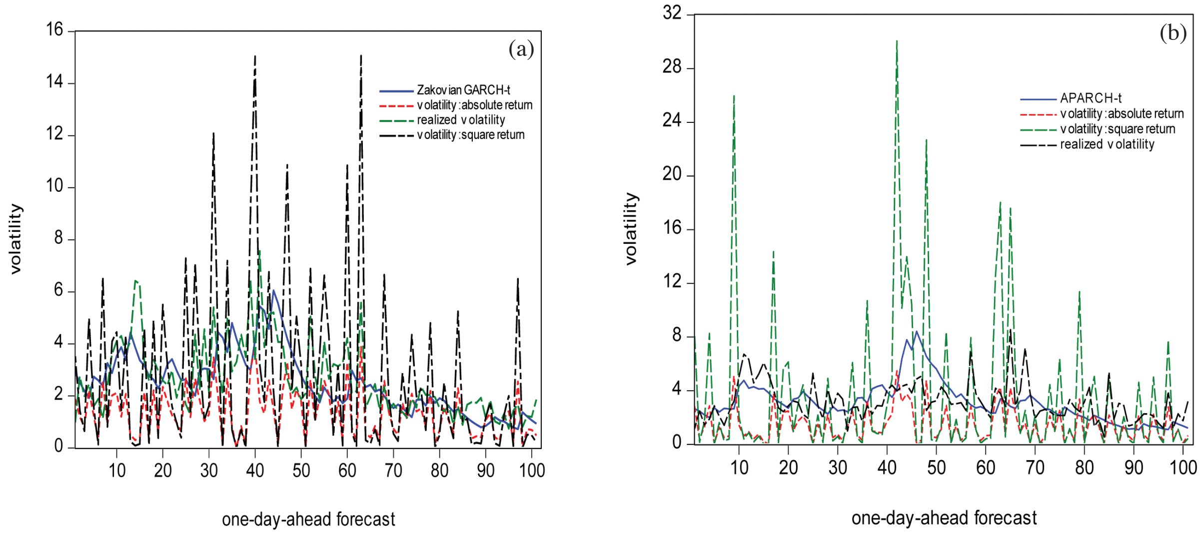

FIGURE 4. Five months volatility forecast results for (a) S&P500 and (b) FTSE100

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